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THE DEMAND FOR MONEY WHEN FIRMS HOLD BALANCES AS PAYMENT FOR SERVICES

David Wiley Mullins, Jr.

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I Introduction

There are three major areas in the study of the demand for money by firms in which economists have been unable to reconcile rational, theoretical models with firm behavior. First, existing theoretical models are inadequate in explaining aggregate firm money demand in time series investigations (Goldfeld, 1973). Secondly, empirical studies have been unable to establish conclusively the presence of the economies of scale in money holdings which theoretical models predict (Meltzer, 1963). Finally, studies of the money holdings of individual firms also fail to support the theoretical models (Homonoff and Mullins, 1972; Miller and Orr, 1967; Orr, 1970; Sprenkle, 1969).

This paper seeks to attack only the third problem area. Specifically, studies of firms are unanimous in the conclusion that firms hold inordinately large levels of demand deposits in comparison with the predictions of the Baumol-Tobin (Sprenkle, 1969) and Miller-Orr (Mullins and Homonoff, 1972; Miller-Orr, 1967; Orr, 1970) models. Some researchers see these results as evidence of suboptimal management and prescribe application of the model as the remedy (Mullins and Homonoff, 1972; Miller and Orr, 1967; Orr 1970). This irrational holding of large amounts of excess cash is surprising for large firms replete with business school trained financial managers.

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Other researchers reject the models on the basis of these findings (Sprenkle, 1969). They suggest that the simple inventory-cost minimization models are inadequate in explaining the complex firm-bank relationship which determines money demand. Specifically, the transactions motive for money holding on which such models are predicated is not the only such motive in the U.S. financial environment. Firms hold deposits to receive bank services, and they hold compensating balances which are often required for loans and lines of credit. They argue that simple optimizing models cannot capture these subtle, heuristic reasons for money holding, which may be irrational but nevertheless are the fruit of years of tradition. The data seem to support such arguments, especially in light of the models' failure in the first two problem areas as well.

Although this paper attempts to provide a framework for the rationalization of all such traditional motives for money holding, its focus is on only one such motive: the practice of firms' providing balances in payment for bank services as opposed to submitting cash payments. In Section II I shall briefly outline the traditional argument that, despite its wide use, this practice is irrational. Then I shall attempt to demonstrate that the holding of compensating balances in lieu of payment for services is, in actuality rational. Since the argument is rooted in the institutional constraint

on the rate of return paid on demand deposits to be equal to zero, Section III describes a world in which no such restriction exists, and banks are free to set r_D, the rate of return paid on demand deposits, at a profit maximizing level. Section IV and V present the body of the argument concerning the rationality of service payment compensating balances. The approach employed includes cost-minimizing (inventory) behavior by firms and profit maximization behavior on the part of banks (and firms) in setting compensating balance requirements. The conclusion notes that this analysis provides a foundation for the rationalization of all the various forms of compensating balance requirements. Also, the analysis, which results in higher predicted money holdings for firms, tends to reconcile theoretical inventory models with the data on firm behavior generated in the studies cited above. This paper attempts to demonstrate that transactions money demand theory is sufficient to explain firm behavior.

II The Irrationality of Holding Balances as Payment for Services

The simple argument that this practice is irrational assumes an equilibrium quantity of services supplied by banks to firms ($S=S^D=S^S$) over some time period at an equilibrium price (p^* where marginal revenue equals marginal cost). The value of services sold is $V(S) = p^*S$. If the firm pays cash, the amount it pays ($V(S)$) is equal to the amount the bank receives ($V(S)$). However, if balances are held in payment, the firm must borrow balances (B) at the current bank loan rate (r_L) such that

2.1 $$r_L B = V(S) \text{ or } B = \frac{V(S)}{r_L}$$

if the cost of these services is to be the same as in the cash payment case. However, when the firm borrows and deposits B, the bank receives only

2.2 $$r_L\ (1 - \delta)\ B < V(S)$$

where $\delta \equiv$ the reserve requirement

from lending these deposits.[1] So, by pledging balances as compensation to the banks for services, the firm appears to pay more than the bank receives; the difference being the required reserves which must be

held behind B. The dead weight loss of these idle required reserves benefits neither party. Therefore, this traditional practice has been called irrational.

The argument given above depends on the equality of the rate the firm must pay to secure B and the rate the bank receives when lending these deposits (less reserves = $B(1-\delta))^2$. However, even if the firm secures the funds at a rate lower than r_L, this dead weight loss which direct payment eliminates, is the basis for the contention that this is an irrational practice.

The irrationality argument holds only when the firm must raise some portions of the required balances. If balances normally held can be pledged, this practice is not irrational. In fact, the situation illustrates the traditional rationalization for (unborrowed) compensating balances (see Guttentag and Davis, 1961, 1962; Hellweg, 1961) as a device to attract deposits. Supplying services for balances that firms would hold anyway is viewed as a practice to circumvent the restriction that $r_D = 0$. The inadequacy of this view lies in its inability to explain firms' holding of balances at a level higher than normal, since any additional deposits held for this purpose incur the dead weight reserve loss. Therefore, such an explanation is of no help in explaining the unrealistically (relative to model predictions) high levels of deposits firms actually hold (see Gutentag and Davis, 1961, 1962 and Mullins, 1973 for a full analysis). So while

compensating balances are often suggested as an explanation of firms' holding high levels of deposits, the traditional explanation of the rationality of compensating balances excludes the holding of balances higher than in the absence of such requirements and admits the irrationality of practices requiring the firm to hold additional balances.

In the following two sections I shall attempt to demonstrate that the fallacy in this argument lies in its failure to consider the economics of money holding; specifically the savings in transaction cost accruing to the firm as a result of holding additional balances.

III Firm Money Demand with No Restrictions on the Rate Paid on Demand Deposits

This section explores the bank-firm relationship when there is no constraint on r_D, the rate paid on deposits by banks.

$L^* \equiv$ the firm's desired short term loan position excluding demand deposits.[3]

$L \equiv$ the expected short term bank loans during the time period.[4]

$L = L^* + M$

$M \equiv$ the expected level of firm demand deposits (which is, of course, under management's control).

r_L the bank loan rate

$r_D \equiv$ rate of return paid by the bank on demand deposits

$C_T \equiv$ the expected cost of managing the short term asset position

$C \equiv$ the expected cost to the firm of managing cash

$C = C_T - r_L L^*$

$T(M) \equiv$ the expected transaction cost incurred by the firm in managing its cash.[5] The only restrictions on this function are the assumptions that:

$$\frac{\partial T}{\partial M} < 0 \qquad\qquad \frac{\partial^2 T}{\partial M^2} > 0$$

Since the selection of M does not effect L^*, the firm's decision involves only C (as opposed to C_T). Specifically the firm seeks to minimize its cost of cash management by finding the M^* that yields the minimum C^*:

3.1a $C_T = r_L L - r_D M + T(M) = r_L L^* + (r_L - r_D)M + T(M)$

3.1b $C = (r_L - r_D)M + T(M)$

$$\frac{\partial C}{\partial M} = 0 = r_L - r_D + \frac{\partial T}{\partial M} \quad \text{and}$$

$$\frac{\partial^2 C}{\partial M^2} = \frac{\partial^2 T}{\partial M^2} > 0 \quad \text{insuring a minimum}$$

So C* is found at the M* where:

3.2 $$r_L - r_D = - \frac{\partial T}{\partial M^*}$$

This is the fundamental condition for an optimum in all the transaction demand inventory models.

Now, to examine the bank's reaction to M* in setting r_D, consider the joint position of the bank and the firm in cash management when firms choose M* which minimizes C:

$\pi \equiv$ the bank's profit from the firm's account as a function of desired M.

3.3 $\pi = r_L (1 - \delta)M - r_D M = (r_L(1 - \delta) - r_D)M$

$J \equiv$ the joint position of the bank and the firm in cash management

3.4 $J = \pi - C^* = -r_L \delta M^* - T(M^*)$

So in calculating the joint position, we are left with the cost to the bank of the reserves and the firm's transaction cost. Traditional analysis errs in omitting this second term.

Suppose now the bank sets r_D so as to maximize this joint return from cash management. The $r_D{}^*$ yielding the maximum J* is found where

$$\frac{\partial J}{\partial r_D} = \left(-r_L \delta - \frac{\partial T}{\partial M^*}\right) \frac{\partial M^*}{\partial r_D} = 0$$

We know that since firms have selected the M* which minimizes C that

$$3.2 \qquad -\frac{\partial T}{\partial M^*} = r_L - r_D$$

So J* is found at $r_D{}^*$ where

$$3.5 \qquad (r_L(1 - \delta) - r_D) \frac{\partial M^*}{\partial r_D} = 0$$

From the firm's cost minimization conditions and the assumptions concerning T(M)

$$3.6 \qquad M^* = \phi\left(-\frac{\partial T}{\partial M}\right) = \phi(r_L - r_D)$$

Where ϕ is the inverse of minus the marginal transaction cost function, $\frac{\partial T}{\partial M}$

Therefore

3.7 $$\phi' \equiv \frac{\partial \phi}{\partial - \frac{\partial T}{\partial M}} = \frac{1}{-\frac{\partial^2 T}{\partial M^2}} < 0$$ and

by the chain rule,

3.8 $$\frac{\partial M^*}{\partial r_D} = \frac{1}{\frac{\partial^2 T}{\partial M^2}} > 0$$

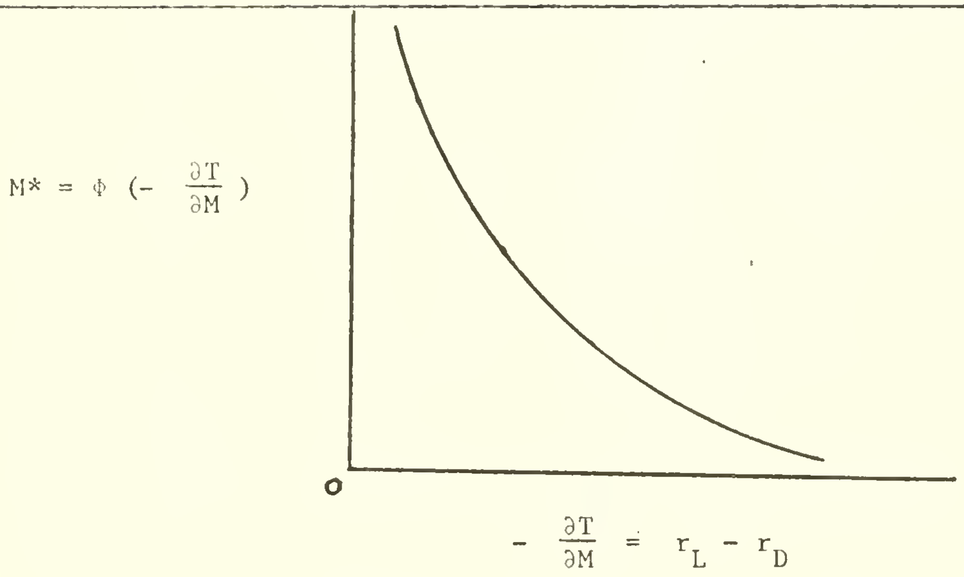

Figure 3.1

Therefore as long as $\frac{\partial^2 T}{\partial M^2} > 0$ (which is necessary to assure a

minimum value of C) then $\frac{\partial M^*}{\partial r_D} > 0$ as one would expect. Hence, J* is found at

3.9 $$r_D^* = r_L(1 - \delta)$$

$$\frac{\partial^2 J}{\partial r_D^2} = (-r_L\delta - \frac{\partial T}{\partial M})\frac{\partial^2 M^*}{\partial r_D^2} - \frac{\partial^2 T}{\partial M^{*2}}\left(\frac{\partial M^*}{\partial r_D}\right)^2$$

and from 3.2 and 3.9 $r_L - r_D^* = r_L\delta = -\frac{\partial T}{\partial M^*}$, therefore

$$\left.\frac{\partial^2 J}{\partial r_D^2}\right|_{r_D = r_D^*} = -\frac{\partial^2 T}{\partial M^{*2}}\left(\frac{\partial M^*}{\partial r_D^*}\right)^2$$ and from 3.8

$$= -\frac{1}{\frac{\partial^2 T}{\partial M^2}} < 0$$ insuring a maximum.

Substituting this value of r_D^* (3.9) into the firm's cost minimization decision yields the actual firm money demand, M**, (ie. the $M^*(r_D)$ when $r_D = r_D^*$) resulting from the optimizing behavior of both firms and banks. So when banks offer the optimum $r_D(r_D^* = r_L(1 - \delta))$, firms will choose M** such that

3.10 $$-\frac{\partial T}{\partial M^{**}} = r_L - r_D^* = r_L\delta$$

This M** defined by 3.10 is the money demand which would be generated in a commercial banking system in which there is no restriction on r_D.

Note that with $r_D < r_L\ (1 - \delta)$, $\frac{\partial J}{\partial r_D} > 0$. Therefore there is an incentive to increase r_D (and M*) since the marginal savings in transaction cost is greater than the profit loss due to required reserves (ie. $-\frac{\partial T}{\partial M} > r_L\delta$). The optimum is found where the marginal savings in transaction cost equals the lost revenue due to reserves. Note also that only at r_D^* is the opportunity cost of M to the firm $(r_L - r_D)$ equal to the bank's opportunity cost on additional deposits $(r_L\delta)$. At lower levels of r_D the firm sees M as more costly than the bank.

It is also interesting to note that this r_D^* is the value resulting from a competitive commercial banking system with no constraints on interest payments on deposits. This is true since in such a system competition drives r_D up to the level where profit equals zero. So

3.11 $$\pi = 0 = (r_L(1 - \delta) - r_D)\ M \Rightarrow r_D = r_L(1 - \delta)$$

In this case as r_D is increased from lower levels the firm receives all the benefits of the resulting savings in transaction cost while the banks must absorb the additional reserve costs. Also in such a world there is no incentive to supply deposits for services, since an increase in M* above M** yields a loss due to reserves which is greater than the corresponding savings to the firm.

Finally, consider a monopolistic banking system where the banker sets r_D to maximize his profit without regard to the firm's situation.

$$\pi = (r_L(1 - \delta) - r_D)M^*$$

$$\frac{\partial \pi}{\partial r_D} = (r_L(1 - \delta) - r_D)\frac{\partial M^*}{\partial r_D} - M^*$$

$$\left.\frac{\partial \pi}{\partial r_D}\right|_{r_D = r_D^* = r_L(1-\delta)} = -M^* < 0$$

So the monopolist would want to set $r_D < r_D^*$. [6]

However, since $\frac{\partial J}{\partial r_D} > 0$ at $r_D < r_L(1 - \delta)$, even a monopolist will desire $r_D{}^* = r_L(1 - \delta)$ (yielding J*), if he can recoup through side-payments etc. an amount greater than (or equal to) his lost profits due to required reserves. If he is all powerful, he can recoup all the firm's savings in transaction cost, leaving the firm no worse (nor better) off and resulting in an increase in returns to the bank of $J^*_{r_D{}^*} - J_{r_D < r_D{}^*} = \Delta J$. For banks possessing such market power, overcharging for services might be a technique to capture

these returns, monopolists will set the same r_D^* as competitive bankers, and hence, firm money demand will be the same (M**) in either situation.

In conclusion of this section and introduction of the next, in a banking system with r_D' constrained to be equal to zero, there exists an incentive to increase deposits in order to reap the benefits of savings in transaction cost. One might expect this to register most strongly in large firms from whose view point the banking system is competitive. In such a situation banks competing for firms' deposits would be expected to develop methods of circumventing the restriction on r_D. Firms in this situation demand this, for they reap the benefits directly. This situation is characteristic of the relationship between large U.S. firms and banks, and it is among such large firms and large banks that the various compensating balance practices are most prevalent. Even banks with market power with respect to firms will desire such practices if they can share in the benefits. However, since the mechanism apparently is not clearly understood by bankers, the counter-intuitive policy of a monopolist increasing r_D would seem unlikely. Hence, it would be less likely that banks with market power over their customers would experiment and discover the benefits of this practice. Also, there is a direct loss to

such banks ($-r_L\delta$), while the benefits are indirect and must be captured through some side-payment mechanism. The conclusions of this section are summarized in Table 3.1.

The next section presents an analysis of one practice designed to reap these benefits: that of holding deposits as payment for bank services.

Table 3.1

Summary of Firm Money Demand with No Restriction on the Rate Paid on Demand Deposits

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Bank Maximization Yields : $r_D^* = r_L(1 - \delta)$

Firm's Cost Minimization Condition: $-\frac{\partial T}{\partial M^{**}} = r_L - r_D^* = r_L\delta$

Yielding Optimal Firm Money Demand: $M^{**} = \Phi(-\frac{\partial T}{\partial M^{**}}) = \Phi(r_L - r_D^*) = \Phi(r_L\delta)$

Where Φ is the inverse of minus the marginal transaction cost function, $-\frac{\partial T}{\partial M}$.

Optimal Firm Money Demand Yields:

Total Interest on Deposits = $r_D^*M^{**} = r_L(1 - \delta)\ (\Phi(r_L\delta))$

$C^{**} = (r_L - r_D^*)M^{**} + T(M^{**}) = (r_L\delta)\ (\Phi(r_L\delta)) + T(\Phi(r_L\delta))$

$\pi^* = (r_L(1 - \delta) - r_D^*)M^{**} = 0$

$J^* = -\ r_L\delta M^{**} - T(M^{**}) = (-\ r_L\delta)\ (\Phi(r_L\delta)) - T(\Phi(r_L\delta))$

IV. Compensating Balances as Payment for Services

This section examines the demand for money by firms when they have the option of supplying balances in payment for some or all of the bank services normally demanded. The firm has a demand for bank services ($p^D(S)$), and the bank has a schedule outlining the unit price it charges for any quantity of services delivered to a firm ($p^S(S)$). It is assumed

$$\frac{\partial p^D}{\partial S} < 0$$

Furthermore, the assumptions

$$\lim_{S \to 0^+} p^S(S) < \lim_{S \to 0^+} p^D(S) \quad \text{and} \quad \left.\frac{\partial p^D}{\partial S}\right|_{S=\overline{S}} < \left.\frac{\partial p^S}{\partial S}\right|_{S=\overline{S}} \quad \text{for any and all } \overline{S}$$

are sufficient to insure an equilibrium.

If the firm pays cash for all services, S*, the equilibrium quantity of services is found at

4.1 $$p^* \equiv p^D(S^*) = p^S(S^*) \qquad \text{(see Figure 4.1)}$$

In addition to selling services for cash, the bank is also willing to supply services as the firm puts up, M, (average) demand deposit.[7]

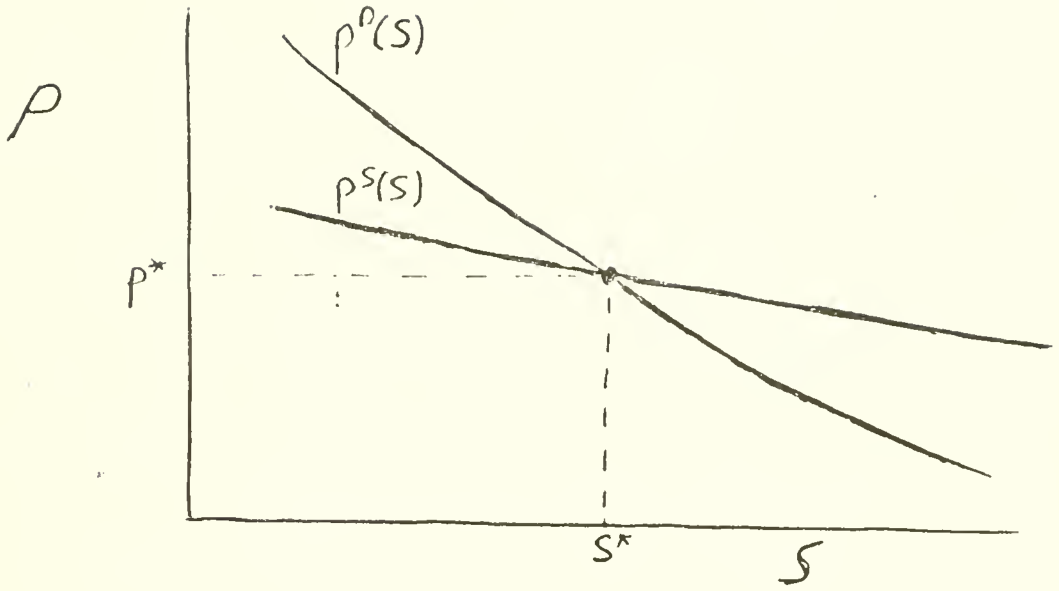

Figure 4.1

4.2a $S_M = S_M(M)$

S_M is the quantity of services the firm receives from holding average deposits of M. It is assumed

$$S'_M \equiv \frac{\partial S_M}{\partial M} > 0 \text{ and } \frac{\partial^2 S_M}{\partial M^2} = 0 \text{ ,and specifically}$$

4.2b $S_M = S'_M M$

S'_M is the bank's decision variable. Later, we shall derive the optimal s'_M and examine its impact on M. So given any M, the firm receives a quantity of services S_M.

It is also assumed in this section

4.3 $S_M \leq S^*$

The firm chooses to supply balances for some or all of the services it would normally demand in the pure cash payment situation. The possibility that this mechanism will expand the firm's total demand for services (ie $S_M > S^*$) is examined in Section V.

Given 4.3, the firm will still receive a total quantity of services equal to S*. Therefore, its unit price is p*. If it paid cash for all of these services its bill would be p*S*. However,

since it receives S_M as a result of holding M, it pays only $P^*(S^* - S_M)$.

$R_F(S_M(M))$ the savings to the firm from receiving S_M services as a result of holding M (ie the reduction in the firm's bill for services). Hence, R_F is equal to the amount the firm would have to pay to the bank to receive S_M in the absence of the compensating balance arrangement.

4.4 $R_F(S_M(M))= p^*S^* - p^*(S^* - S_M) = p^*S_M$ for $S_M \leq S^*$

Therefore, when $S_M \leq S^*$ the total services demanded and supplied, S^*, remains the same as in the pure cash payment situation, and it is this total quantity which determines the applicable unit price, p^*. However, with compensating balances, the firm receives S_M as a result of holding M and pays $p^*(S^* - S_M)$ in cash only for the additional services $S^* - S_M$.

$R_B(S_M(M)) \equiv$ the reduction in revenue from the bank's viewpoint resulting from supplying services of S_M for deposits of M.

4.5 $R_B(S_M(M)) = p^*S^* - p^*(S^* - S_M) = p^*S_M$ for $S_M \leq S^*$

Hence, with compensating balances the firm still demands total services of S^*, and its savings in service charges from receiving S_M(ie R_F) is equal to the bank's loss in revenue providing S_M.

Note that

4.6 $$R'_F \equiv \frac{\partial R_F}{\partial S_M} = p^* > 0 \quad \text{and}$$

4.7 $$\frac{\partial R'_F}{\partial S_M} = \frac{\partial^2 R_F}{\partial S_M^2} = 0 \qquad \text{for } S_M \leq S^*$$

Also

4.8 $$\frac{\partial R_F}{\partial M} = \frac{\partial R_F}{\partial S_M}\frac{\partial S_M}{\partial M} = R'_F S'_M > 0 \quad \text{and}$$

4.9 $$\frac{\partial^2 R_F}{\partial M^2} = \frac{\partial^2 R_F}{\partial {S_M}^2}\left(\frac{\partial S_M}{\partial M}\right)^2 + \frac{\partial R_F}{\partial S_M}\frac{\partial^2 S_M}{\partial M^2} = 0 \qquad \text{for } S_M \leq S^*$$

$\frac{\partial R_F}{\partial M}$ is interpreted as the marginal savings in service charges resulting from a small increment in the firm's average balance.

As before,

$T(M) \equiv$ the expected transaction cost as a function of the expected level of deposits,

and it is assumed

$$\frac{\partial T}{\partial M} < 0 \quad \text{and} \quad \frac{\partial^2 T}{\partial M^2} > 0.$$

Furthermore, we shall assume that in this economy the deposit rate is institutionally constrained to be equal to zero. So

4.10 $$r_D = 0$$

As before since L* is not affected by M, the firm's decision consists of minimizing the cost of cash management.

4.11 $$C_T = r_L L - r_D M + T(M) - R_F(S_M(M))$$

$$= r_L L^* + r_L M + T(M) - R_F(S_M(M))$$

$$C = C_T - r_L L^*$$

4.12 $$C = r_L M + T(M) - R_F(S_M(M))$$

$$\frac{\partial C}{\partial M} = r_L + \frac{\partial T}{\partial M} - \frac{\partial R_F}{\partial M} = r_L + \frac{\partial T}{\partial M} - \frac{\partial R_F}{\partial S_M}\frac{\partial S_M}{\partial M} = 0$$

$$\frac{\partial^2 C}{\partial M^2} = \frac{\partial^2 T}{\partial M^2} - \frac{\partial^2 R_F}{\partial M^2} > 0$$ insuring a minimum.

So to achieve C*, the minimum C, the firm would choose M* such that

4.13 $$r_L - \frac{\partial R_F}{\partial M^*} = r_L - \frac{\partial R_F}{\partial S_M}\;\frac{\partial S_M}{\partial M^*} = -\frac{\partial T}{\partial M^*}$$

Note that the savings (ie reduction in service cost) received for a marginal increase in M, $\frac{\partial R_F}{\partial M}$, plays a role similar to cash r_D in the previous analysis. It is a "shadow" rate of return on demand deposits from the firm's viewpoint. For convenience, we shall call this implicit rate

4.14 $$r_S \equiv \frac{\partial R_F}{\partial M} = R'_F S'_M$$

To illustrate the bank's behavior we shall examine π, the bank's expected profit from the firm's account.

4.15 $$\pi = r_L(1 - \delta)M - r_D M - R_B(S_M(M)) \quad \text{and since}$$

$$\pi = r_L(1 - \delta)M - R_B(S_M(M))$$

As before, consider the joint position of the bank and the firm in cash management when firms choose M* $r_D=0$ minimizes C.

4.16 $$J \equiv \pi - C^* = r_L \delta M^* - T(M^*) + R_F(S_M(M^*)) - R_B(S_M(M^*))$$

From 4.4 and 4.5

4.17 $$R_F(S_M(M)) = R_B(S_M(M)) \qquad \text{for } S_M \leq S^*$$

Therefore

4.18 $$J = -r_L\delta M^* - T(M^*) \qquad \text{for } S_M \leq S^*$$

The bank affects J by setting S'_M, the rate at which the bank supplies services for deposits, which determines r_S, the shadow rate of return on deposits. Then , r_S determines M through the firm's cost minimization decision, 4.13.

To find J*, the maximum J

$$\frac{\partial J}{\partial S'_M} = 0 = (-r_L\delta - \frac{\partial T}{\partial M^*})\frac{\partial M^*}{\partial S'_M} = (-r_L\delta - \frac{\partial T}{\partial M^*})\frac{\partial M^*}{\partial r_S}\frac{\partial r_S}{\partial S'_M}$$

See the appendix for the derivation of

4.19 $$\frac{\partial M^*}{\partial r_S} = \frac{1}{\frac{\partial^2 T}{\partial M^{*2}}} > 0 \quad \text{and}$$

4.20 $$\frac{\partial r_S}{\partial S'_M} = \frac{R'_F}{1 - S'_M \frac{\partial R'_F}{\partial S_M}\frac{\partial S_M}{\partial M^*}\frac{\partial M^*}{\partial r_S}} = \frac{\frac{\partial R_F}{\partial S_M}\frac{\partial^2 T}{\partial M^{*2}}}{\frac{\partial^2 T}{\partial M^{*2}} - \frac{\partial^2 R_F}{\partial S_M^2}(\frac{\partial S_M}{\partial M^*})^2} > 0$$

Recall that firms are minimizing C, so they choose M* such that

4.13 $$-\frac{\partial T}{\partial M^*} = r_L - r_S = r_L - R_F'S_M'$$

Substituting,we find

$$\frac{\partial J}{\partial S_M'} = (r_L(1 - \delta) - R_F'S_M')\ \frac{\partial M^*}{\partial r_S}\ \frac{\partial r_S}{\partial S_M'} = 0$$

So to achieve J*, the bank sets S_M' such that

4.21 $$(R_F'S_M')^* = r_S^* = r_L(1 - \delta)$$

(Also, from 4.17 $(R_B'S_M')^* = r_S^* = r_L(1 - \delta)$)

Therefore

4.22 $$S_M'^* = \left(\frac{\partial S_M}{\partial M^*}\right)^* = \frac{r_L(1 - \delta)}{\dfrac{\partial R_F}{\partial S_M}} = \frac{r_L(1 - \delta)}{\dfrac{\partial R_B}{\partial S_M}} = \frac{r_L(1 - \delta)}{p^*}$$

Also

$$\left.\frac{\partial^2 J}{\partial S_M'^2}\right|_{r_S = r_S^*} = -\frac{\left(\dfrac{\partial r_S}{\partial S_M'}\right)^2}{\dfrac{\partial^2 T}{\partial M^2}} < 0 \text{ from 4.20 and}$$

the assumption that $\frac{\partial^2 T}{\partial M^2} > 0$. This insures a maximum.

Notice that

$$\frac{\partial J}{\partial r_S} = (-r_L \delta - \frac{\partial T}{\partial M^*}) \frac{\partial M^*}{\partial r_S}$$

Since firms are optimizing $-\frac{\partial T}{\partial M^*} = r_L - r_S$, therefore

$$\frac{\partial J}{\partial r_S} = (r_L(1 - \delta) - r_S) \frac{\partial M^*}{\partial r_S}$$

So

4.24 $$\frac{\partial J}{\partial r_S} > 0 \quad \text{for } r_S < r_L(1 - \delta) = r_S^* = r_D^*$$

This implies that with r_D institutionally constrained to be zero and no compensating balance arrangement (ie. $r_S = 0$, see Table 4.1), firms and banks, taken together, are less well off than in the compensating balance case. This is true in spite of the fact that, in the absence of side payments, the bank's profit is lower with compensating balances. Also,

4.25 $$\Phi(r_L) < \Phi(r_L - r_S^*) = \Phi(r_L\delta)$$

Then, optimal money demand with compensating balances, $M^{**} = \Phi(r_L\delta)$ is greater than optimal money demand in the absence of compensating balances (ie. $r_S = 0$) and $r_D = 0$ (ie. in this case $M^{**} = \Phi(r_L)$). Hence, the traditional argument is fallacious, and compensating balances are rational even though they result in higher firm balances.

Thus, such an arrangement is rational. The points raised in part III concerning competitive and monopolistic banking systems apply in this case as well. The critical point in this situation as well is that when the return on deposits is constrained to be below the competitive solution, there exists a strong incentive to develop practices which increase M. In this case as well as before, the savings in transaction cost ($-\frac{\partial T}{\partial M} = r_L - r_S$) generated by additional deposits is greater than the profit loss due to required reserves. Hence, in an economy with $r_D = 0$, both the bank and the firm can be made better off by a practice such as the one analyzed in this section which encourages firms to hold higher balances. The division of the resulting gains depends upon the distribution of power in the bank-firm relationship. It is clear that the traditional argument that this practice is irrational is fallacious. It considers only the bank's

profit loss and ignors the savings in transaction cost which is larger than bank losses. The restriction on r_D in our economy necessitates the introduction of such compensating balance practices to achieve the optimal money demand.

Another issue is how changes in the cost of services effect money demand. One would expect that with the reduction in information processing costs (ie. due to advancements in computing technology), the marginal cost of providing many bank services has declined markedly. Because deposits are held as payment for services, one might expect this to effect the firm's money demand. According to my analysis, such cost changes might effect $p^S(S)$, p^* (and therefore R_F and R_B) and S^*, the equilibrium quantity of services supplied. Therefore, the optimal S'_M would also be effected (ie. since $S'^*_M = \frac{r_L(1 - \delta)}{p^*}$). However, it is the product $R'_F S'^*_M = R'_B S'^*_M = r^*_S$ which determines money demand. Furthermore, if r^*_S is set in accordance with the optimization outlined above, it is constrained to be equal to $r_L(1 - \delta)$ to achieve the optimal position. Hence, with a change in the cost of services, the change in $S'_M = \frac{r_L(1 - \delta)}{p^*}$ (ie. due to a change in P^*) is offset by a change in $R'_F = R'_B = p^*$ leaving r^*_S and the optimal money demand unchanged.

Finally, an important result of this analysis is that the firm's optimal demand for money, M^{**} generated by r^*_S, is the same in an

economy with a competitively set r_D (ie. Section III of this paper) and one with the institutional constraint $r_D = 0$ and the practice of holding balances as payment for services (ie. Section IV). When the optimal rate at which the bank supplies services S'^{*}_{M} (which yields $r_S{}^{*} = r_L(1 - \delta)$) is substituted into the firm's cost minimization decision, the firm will choose an optimal money demand, M**, such that

$$4.26 \qquad -\frac{\partial T}{\partial M^{**}} = r_L - r^{*}_{S} = r_L - R'_F S'_M{}^{*} = r_L \delta \quad \text{from 4.13 and 4.21}$$

This is the same condition for M** as 3.10 in the previous section. The firm chooses the M which equates the marginal reduction in transaction cost resulting from additional balances to $r_L\delta$, the bank's marginal reserve cost. This is the same rule and therefore the same M** that was generated by the analysis in Section III where r_D is unconstrained. So firm money holdings are the same when banks pay explicit interest (r^{*}_{D}) and when it pays implicit interest (r^{*}_{S}) by providing services for balances.

This result is intuitive. The considerations which lead banks to pay implicit interest through services at the rate r^{*}_{S} are the same as those which lead them to pay r^{*}_{D} when explicit interest is allowed. Regardless of whether the interest is paid in cash or services, the optimal money demand is the same.

Hence, because of the economics of money holding and the optimizing behavior of firms and banks, optimally set compensating balance arrangements circumvent the government's restriction on the deposit rate and nullify its impact on money demand. This does not mean, however, that banks and firms are equally well off in the two situations. To be persuaded to enter into such a compensating balance arrangement, banks with monopoly power have to be reimbursed for their reserve losses. Hence, firms have to give to the bank some of the savings in transaction cost, and therefore the banks retain the extra profit created by the restriction in r_D. However, in a competitive banking system, competition for balances forces banks to incur the reserve losses resulting from higher balances, and firms receive all of the savings in transaction costs. Hence competition forces banks to give up the profit cushion provided by the restriction on r_D. Thus, a competitive banking system with optimal compensating balance arrangements results in money demand, cash management costs, and bank profits identical to those in a system with no constraint on the interest rate on demand deposits. This can be seen by comparing the results of the analysis in this section, summarized in Tables 4.1 and 4.2 with those of the preceding section which are outlined in Table 3.1.

Table 4.1

Summary of Firm Money Demand with No Compensating Balance Arrangement and

$r_D = 0$

Institutional Constraint on Bank Yields: $r_D = 0$

Firm's Cost Minimization Condition: $-\frac{\partial T}{\partial M^*} = r_L - r_D = r_L$

Yielding Optimal Firm Money Demand: $M^{**} = \Phi\left(-\frac{\partial T}{\partial M^{**}}\right) = \Phi(r_L - r_D) = \Phi(r_L)$

Where Φ is the inverse of minus the marginal transaction cost function, $-\frac{\partial T}{\partial M}$.

Optimal Firm Money Demand Yields:

Total Interest on Deposits $= r_D M^{**} = 0$

$C^{**} = (r_L - r_D)\, M^{**} + T(M^{**}) = r_L\ \Phi(r_L) + T(\Phi(r_L))$

$\pi^* = (r_L(1 - \delta) - r_D)M^{**} = (r_L(1 - \delta))\ (\Phi(r_L))$

$J^* = -\ r_L\delta\ M^{**} - T(M^{**}) = (-\ r_L\delta)\ (\Phi(r_L)) - T(\Phi(r_L))$

Table 4.2

Summary of Firm Money Demand with Compensating Balances as Payment for Services and $S_M \leq S^*$

$S_M(M) = S'_M M$ $\qquad$ $R_F(S_M(M)) = R_B(S_M(M)) = p^*S_M = p^*S'_M M$

Bank Maximization Yields: $S'^*_M = \frac{r_L(1 - \delta)}{p^*}$ and $r^*_S = R'_F S'^*_M = R'_B S'^*_M = r_L(1-$

Firm's Cost Minimization Condition: $-\frac{\partial T}{\partial M^{**}} = r_L - R'_F S'^*_M = r_L - r^*_S = r_L\delta$

Yielding Optimal Firm Money Demand: $M^{**} = \Phi(-\frac{\partial T}{\partial M^{**}}) = \Phi(r_L - r^*_S) = \Phi(r_L\delta)$

Where Φ is the inverse of minus the marginal transaction cost function, $-\frac{\partial T}{\partial M}$.

Optimal Firm Money Demand Yields:

Total Implicit Interest $= R_F(S_M(M^{**})) = P^*(S_M(M^{**})) = r_L(1 - \delta)(\Phi(r_L\delta))$

$C^{**} = r_L M^{**} - R_F(S_M(M^{**})) + T(M^{**}) = (r_L\delta)(\Phi(r_L\delta)) + T(\Phi(r_L\delta))$

$\pi^* = r_L(1 - \delta)M^{**} - R_B(S_M(M^{**})) = 0$

$J^* = - r_L\delta M^{**} - T(M^{**}) = (-r_L\delta)(\Phi(r_L\delta)) - T(\Phi(r_L\delta))$

Table 4.2 Continued

Comparisons

	No Compensating Balance Arrangement and $r_D=0$ (Table 4.1)		Compensating Balance Arrangement with $S_M \le S^*$ (Table 4.2)		No Compensating Balance Arrangement and r_D unconstrained (Table 3.1)
Interest Rate on Deposits	$r_D(=0)$	$<$	r_S^*	$=$	r_D^*
Optimal Firm Money Demand	M^{**}	$<$	M^{**}	$=$	M^{**}
Optimal Firm Cost	C^{**}	$>$	C^{**}	$=$	C^{**}
Optimal Bank Profit	π^*	$>$	π^*	$=$	π^*
Optimal Bank-Firm Position	J^*	$<$	J^*	$=$	J^*

V. Money Demand When Compensating Balance Requirements Increase the Quantity of Services Demanded by the Firm and the Motivation for Multiple Bank Service Products

In the preceding section it was assumed that $S_M \leq S^*$; that is the quantity of services supplied for balances is less than or equal to the equilibrium quantity the firm would normally purchase. We derived $S_M'^*$, the optimal rate at which the bank supplies services for deposits, and r_S^*, the optimal "shadow" or implicit rate of return on deposits. This r_S^* results in M^{**}, the optimal money demand. $S_M = S_M'^* M^{**}$ is the quantity of services the firm receives when it holds M^{**}. It is conceivable that S_M calculated in this manner is greater than S^*, the equilibrium quantity the firm would purchase in the absence of compensating balances. If this is the case, the analysis in Section IV is no longer valid. In this section we examine this possibility by assuming

5.1 $\quad S_M > S^*$

If 5.1 holds, the quantity of services the firm receives from holding M is greater than the amount it would demand in the absence of compensating balances. The amount it would be willing to pay for this quantity of services is $(p^D(s_M))(S_M)$. Therefore, the savings

to the firm in service charges from receiving S_M for holding M is

5.2 $$R_F(S_M(M)) = (p^D(S_M(M)))(S_M(M)) \quad \text{for } S_M > S^*$$

Note that the savings from the point of view of the firm associated with receiving S_M when $S_M > S^*$ is only the amount it would be willing to pay for these services. It is not the (larger) amount it would have to pay (ie.$(p^S(S_M))$ $(S_M))$ to purchase S_M, since it would be unwilling to pay this amount.

Similarly in this case

5.3 $$R_B(S_M(M)) = (p^S(S_M(M)))(S_M(M)) \quad \text{for } S_M > S^*$$

From the bank's viewpoint, its lost revenue from providing $S_M > S^*$ is $(p^S(S_M))(S_M)$, since that is what it would charge to provide this quantity of services.

Note that from the preceding section

4.17 $$R_F(S_M(M)) = R_B(S_M(M)) \quad \text{for } S_M \leq S^*$$

Now

5.4 $$R_F(S_M(M)) < R_B(S_M(M)) \quad \text{for } S_M > S^* \text{ (see Figure 5.1)}$$

Hence, as long as $S_M \leq S^*$, the firm still demands total services of S^*, and its savings from receiving S_M is equal to the bank's revenue loss from providing S_M. But when this mechanism results in a quantity of services supplied for balances which is greater than the normal equilibrium quantity (ie. $S_M > S^*$), the amount the bank would charge for this quantity is greater than the amount the firm would be willing to pay. Hence 5.4 holds: the reduction in revenue from the bank's viewpoint is greater than the cost savings from the point of view of the firm.

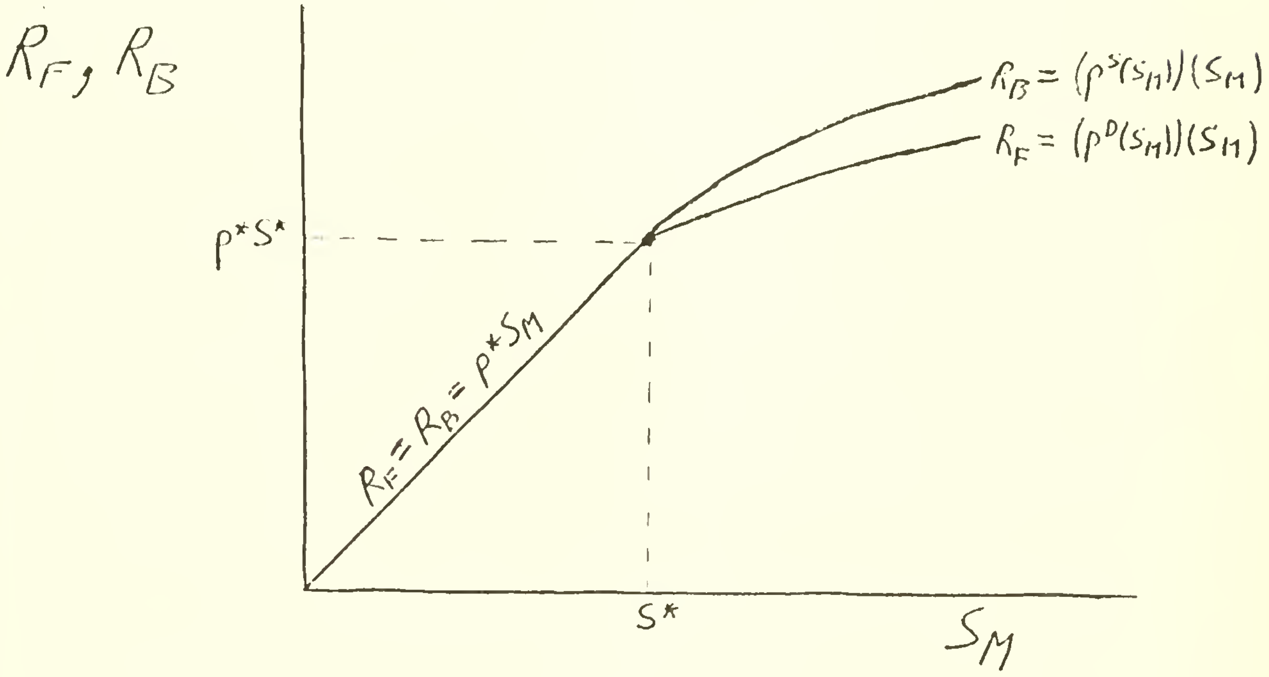

Figure 5.1

To insure $r_S > 0$, It is necessary to assume

5.5 $$R'_F \quad \frac{\partial R_F}{\partial S_M} > 0 \qquad \text{for } S_M > S^*$$ [8]

and it is assumed

5.6 $$\frac{\partial R'_F}{\partial S_M} = \frac{\partial^2 R_F}{\partial S_M^2} \leq 0 \qquad \text{for } S_M > S^*$$

as before

5.7 $$\frac{\partial R_F}{\partial M} = \frac{\partial R_F}{\partial S_M} \frac{\partial S_M}{\partial M} = R'_F S'_M = r_S > 0 \text{ for } S_M > S^*$$ [9]

but now

5.8 $$\frac{\partial^2 R_F}{\partial M^2} = \frac{\partial^2 R_F}{\partial S_M^2} \left(\frac{\partial S_M}{\partial M} \right)^2 + \frac{\partial R_F}{\partial S_M} \frac{\partial^2 S_M}{\partial M^2} = \frac{\partial r_S}{\partial M} \leq 0$$

As before concerning T(M),

$$\frac{\partial T}{\partial M} < 0 \text{ and } \frac{\partial^2 T}{\partial M^2} > 0 \qquad \text{and}$$

as in the preceding section

5.9 $$r_D = 0$$

Again

5.10a $$C_T = r_L L^* + r_L M + T(M) - R_F(S_M(M))$$

$$C = C_T - r_L L^*$$

5.10b $$C = r_L M + T(M) - R_F(S_M(M))$$

Minimizing cost as before, we find C* at M* such that

5.11 $$r_L - \frac{\partial R_F}{\partial M^*} = r_L - R_F' S_M' = r_L - r_S = - \frac{\partial T}{\partial M^*}$$

Where $r_S = \frac{\partial R_F}{\partial M^*}$, the marginal savings resulting from a small increment in M, is again the shadow rate on deposits from the firm's viewpoint. The sufficient condition for a minimum is satisfied since, from our assumptions,

$$\frac{\partial^2 C}{\partial M} = \frac{\partial^2 T}{\partial M^2} - \frac{\partial R_F'}{\partial S_M} (S_M')^2 + R_F' \frac{\partial S_M'}{\partial M} = \frac{\partial^2 T}{\partial M^2} - \frac{\partial R'_F}{\partial S_M} (S_M')^2 > 0$$

Again

5.12 $$\pi = r_L(1 - \delta)M - R_B(S_M(M))$$

5.13 $$J = \pi - C^* = -r_L \delta M^* - T(M^*) + R_F(S_M(M^*)) - R_B(S_M(M^*))$$

and now from 5.2 and 5.3

5.14 $J = -r_L\delta M^* - T(M^*) + (p^D(S_M) - p^S(S_M))(S_M(M^*))$ for $S_M > S^*$

The bank minimizes J by determining S'_M

$$\frac{\partial J}{\partial S'_M} = (-r_L\delta - \frac{\partial T}{\partial M} + R'_F S'_M - R'_B S'_M)\frac{\partial M}{\partial r_S}\frac{\partial r_S}{\partial S'_M}$$

and substituting for $-\frac{\partial T}{\partial M}$ from 5.11

$$\frac{\partial J}{\partial S'_M} = (r_L(1-\delta) - R'_B S'_M)\frac{\partial M}{\partial r_S}\frac{\partial r_S}{\partial S'_M}$$

Since it is demonstrated in the appendix that $\frac{\partial M}{\partial r_S}\frac{\partial r_S}{\partial S'_M} > 0$

J* is found at

5.15 $(r'_B S'_M)^* = r_L(1-\delta)$

(Note that $(R'_B S'_M)^* = r_L(1-\delta) > R'_F S'_M = r_S$ from 5.4)

or

5.16a $S'_M{}^* = (\frac{\partial S_M}{\partial M^*})^* = \frac{r_L(1-\delta)}{\frac{\partial R_B}{\partial S_M}} = \frac{r_L(1-\delta)}{R'_B} = \frac{r_L(1-\delta)}{p^S + \frac{\partial p^S}{\partial S}S_M}$

Unfortunately, the sign of $\frac{\partial^2 J}{\partial S'_M{}^2}$ is no longer unambiguous.

Therefore, we are not guaranteed that 5.16a will yield a maximum value of J. Furthermore, restating 5.16a in a more specific functional form

5.16b $$S_M'^* = \frac{r_L(1-\delta)}{R_B'(S_M(M^*(r_S(S_M'^*))))}$$

(where all the parentheses in the denominator denote functions)

Thus, the bank must solve this function for $S_M'^*$ yielding r_S^* which determines the optimal money demand, M^{**}. In addition the resulting M^{**} must satisfy

5.17 $$S_M(M^{**}) = S_M'^* M^{**} > S^*$$ [10]

Assuming that a solution, $S_M'^*$, satisfying 5.16b and 5.17 exists and yields a maximum value of J, then

5.18 $$r_S^* = R_F' S_M'^* = \frac{R_F'}{R_B'} r_L(1-\delta) = \left(\frac{p^D + \frac{\partial p^D}{\partial S_M} S_M}{p^S + \frac{\partial p^S}{\partial S_M} S_M} \right) r_L(1-\delta)$$

and

5.19 $$\frac{p^D + \frac{\partial p^D}{\partial S_M} S_M}{p^S + \frac{\partial p^S}{\partial S_M} S_M} < 1$$

This follows from the assumptions (in the begining of Section IV) which provide sufficient conditions for an equilibrium in the market for services (as well as being quite reasonable assumptions concerning the form of the supply and demand functions). Specifically, 5.19 results from the assumption that $\left.\frac{\partial p^D}{\partial S}\right|_{S=\bar{S}=S_M} < \left.\frac{\partial p^S}{\partial S}\right|_{S=\bar{S}=S_M}$ for any and all $\bar{S}$, and the fact that, since at S^*, $p^S = p^D$; then at $S=S_M>S^*$, $p^S>p^D$.

Therefore,

5.20 with $S_M>S^*$, $r_S^* < r_L(1-\delta) = r_S^*$ with $S_M \leq S^*$

and optimal money demand with $S_M > S^*$ is

5.21 $$M^{**} = \Phi(r_L - r_S^*) = \Phi\left(r_L - \frac{R_F'}{R_B'} r_L(1-\delta)\right)$$

Since $\frac{\partial M}{\partial r_S} > 0$, with $S_M > S^*$, $M^{**} < M^{**}$ with $S_M \leq S^*$(or with explicit deposit rate $r_D^* = r_L(1-\delta)$).

Hence, the implicit (marginal) interest rate and optimal money demand are less in this case than in the previous two cases. This is because the compensating balance arrangement has created a total quantity of services which is larger than the normal equilibrium quantity. Recall that with $S_M < S^*$, the firm still receives total services of S^*: $S_M(M^{**})$ through holding balances of M^{**} and $S^* - S_M$

by direct cash payment. The expansion of services delivered when $S_M > S^*$ creates a dead weight loss.[11] The bank perceives the implicit (marginal) deposit rate as $R'_B S'_B$ and as before, equates this rate to $r_L(1-\delta)$. However, the firm values S_M at the unit price it would be willing to pay (ie. $p^D(S_M)$) for this quantity. This price is less than $p^S(S_M)$, the unit price at which the bank is willing to sell S_M. Thus, the firm perceives its optimal implicit (marginal) deposit rate as $R'_F S'^*_M < R'_B S'^*_M$. The firm's total implicit interest received, R_F, and its average implicit deposit rate, $p^D S'^*_M = \frac{R_F}{M^{**}}$ (which is strictly greater than its marginal rate when $S_M > S^*$), are also less than the total implicit interest paid, R_B, and the bank's average deposit rate $p^S S'^*_M = \frac{R_B}{M^{**}}$, respectively.

Furthermore, recall that in general

$$5.22a \quad J^* = \pi^* - C^{**} = -\, r_L \delta M^{**} - T(M^{**}) + R_F(S_M(M^{**})) - R_B(S_M(M^{**}))$$

where $M^{**} = \Phi(r_L - r^*_S)$ or

$$5.22b \quad J^* = A(M^{**}(r^*_S)) + R_F(S_M(M^{**}(r^*_S))) - R_B(S_M(M^{**}(r^*_S)))$$

where

$$5.23 \quad A(M^{**}(r^*_S)) \equiv -\, r_L \delta M^{**} - T(M^{**})$$

and

$$\frac{\partial A}{\partial r_S^*} = (-r_L\delta - \frac{\partial T}{\partial M^{**}}) \frac{\partial M^{**}}{\partial r_S^*}$$

Since firms are optimizing, $-\frac{\partial T}{\partial M^{**}} = r_L - r_S^*$, so

$$\frac{\partial A}{\partial r_S^*} = (r_L(1-\delta) - r_S^*) \frac{\partial M^{**}}{\partial r_S^*}$$

Since $\frac{\partial M^{**}}{\partial r_S^*} > 0$ from 4.19 and with $S_M > S^*$, $r_S^* = \frac{R_F'}{R_B'} r_L(1-\delta) < r_L(1-\delta)$ from 5.20, therefore

$$\left.\frac{\partial A}{\partial r_S^*}\right|_{S_M > S^*} > 0$$

Recall that with $S_M \leq S^*$ $r_S^* = r_L(1 - \delta) = r_D^*$ with an explict cash deposit rate

Hence,

5.24 with $S_M > S^*$ $A(M^{**}(r_S^*)) < A(M^{**}(r_S^*))$ With $S_M \leq S^*$

and from 5.4 $R_F < R_B$ when $S_M > S^*$ and from 4.17 $R_F = R_B$ when $S_M \leq S^*$; therefore

5.25 $$J^*\Big|_{S_M > S^*} < J^*\Big|_{S_M \leq S^*}$$

So when $S_M > S^*$, r_S^*, the firm's implicit (marginal) deposit rate, M^{**}, the optimal firm money demand, and J^* are less than their values in the cases analyzed in Sections III and IV. This results from the fact that S_M is not an equilibrium quantity of services in that $p^D(S_M) \neq p^S(S_M)$. This has two results. First, the implicit deposit rate (from the firm's viewpoint) is less than the optimal rate in the previous two cases resulting in lower values of M^{**} and $A(M^{**}(r_S^*))$ (see 5.23). Secondly, a dead weight loss benefiting neither party is created in that instead of being equal (as in Section IV with $S_M \leq S^*$), now $R_F < R_B$. These two factors are responsible for the lower value of J^* in this case.

These results, summarized in Table 5.1, imply that when $S_M > S^*$ the firm and bank, taken together, are less well off than in the previous cases. However, this does not necessarily mean that compensating balances are irrational in this case. In the absence of a compensating balance arrangement (ie. $r_S = 0$) and with $r_D = 0$ (see Table 4.1),

5.26 $$M^{**} = \Phi(r_L) < \Phi(r_L - r_S^*) = M^{**} \text{ with } S_M > S^* \text{ and}$$

5.27 $$A(\Phi(r_L)) < A(\Phi(r_L - r^*_S))$$

The inequalities follow from the fact that $\frac{\partial M}{\partial r_S} > 0$, $\frac{\partial A}{\partial r_S} > 0$ and with $S_M > S^*$, $r^*_S = R'_F S'^*_M > 0$ from 5.7. The difference in A in 5.27 results from the fact that with $S_M > S^*$ the firm is receiving a positive amount of implicit interest in services as compared to none in the absence of compensating balances and with $r_D = 0$.

So, compensating balances are rational even with $S_M > S^*$ when

5.28 $$J^*\Big|_{r^*_S = r_D = 0} < J^*\Big|_{r^*_S = \frac{R'_F}{R'_B} r_L(1-\delta) > 0}$$

which is true if and only if

5.29 $$A(\Phi(r_L - r^*_S)) - A(\Phi(r_L)) > R_B(S_M(\Phi(r_L - r^*_S))) - R_F(S_M(\Phi(r_L - r^*_S)))$$

where $r^*_S = \frac{R'_F}{R'_B} r_L(1-\delta)$

Hence, when compared to the case with no compensating balances and $r_D = 0$, unless this difference in A is totally offset by the dead weight loss term, $R_F - R_B$, the firm and bank, taken together, are better off under the compensating balance arrangement when $S_M > S^*$[12].

However, as demonstrated in 5.25, they are not so well off as in either the explicit interest payment case or the case in which $S_M \leq S^*$. There is a way to avoid the difficulties created by the fact that $S_M > S^*$. So far in the analysis, bank services have been treated as homogenous. Actually, we have been describing the market for a single bank product. Consider the more realistic assumption that the bank provides N different services (or products) denoted S_i, i = 1 to N. Also as before, in a compensating balance arrangement, the bank supplies $S_{Mi}(M)$ in return for firm deposits of M. Again S'_{Mi}, the rate at which the bank supplies each service product, is the bank's decision variable. Using our previous notation and assumptions

$$5.30 \quad C = r_L M + T(M) - \sum_i^N R_{Fi}(S_{Mi}(M))$$

and

$$\frac{\partial C}{\partial M} = 0 = r_L + \frac{\partial T}{\partial M} - \sum_i^N \frac{\partial R_{Fi}}{\partial S_{Mi}} \frac{\partial S_{Mi}}{\partial M} = r_L + \frac{\partial T}{\partial M} - \sum_i^N R'_{Fi} S'_{Mi}$$

So C* is found at M* where

$$5.31 \qquad -\frac{\partial T}{\partial M^*} = r_L - \sum_i^N R'_{Fi} S'_{Mi} = r_L - \sum_i^N r_{Si}$$

and $\frac{\partial^2 C}{\partial M^2} > 0$ if our previous assumptions are applied to the markets for all S_i.

5.32 $$\pi = r_L(1-\delta)M - \sum_{i}^{N} R_{Bi}(S_{Mi}(M))$$

Assuming for the moment that $S_{Mi} \leq S^*_i$ for all i, then from 4.17 $R_{Fi} = R_{Bi}$ for all i and

5.33 $$J = \pi - C^* = -r_L\delta M^* - T(M^*)$$

J* is found at the optimal set of S'^*_{Mi}, i = 1 to N, which jointly satisfy the following system of equations

5.34 $$\begin{aligned} \frac{\partial J}{\partial S'_{M1}} &= \left(-r_L\delta - \frac{\partial T}{\partial M^*}\right)\frac{\partial M^*}{\partial S'_{M1}} = 0 \\ &\vdots \\ \frac{\partial J}{\partial S'_{Mi}} &= \left(-r_L\delta - \frac{\partial T}{\partial M^*}\right)\frac{\partial M^*}{\partial S'_{Mi}} = 0 \\ &\vdots \\ \frac{\partial J}{\partial S'_{MN}} &= \left(-r_L\delta - \frac{\partial T}{\partial M^*}\right)\frac{\partial M^*}{\partial S'_{MN}} = 0 \end{aligned}$$

As before, $\frac{\partial M}{\partial S'_{Mi}} = \frac{\partial M}{\partial r_{Si}} \frac{\partial r_{Si}}{\partial S'_{Mi}} > 0$ for all i and since firms are optimizing we can substitute for $-\frac{\partial T}{\partial M}$ from 5.31. Hence the equations are satisfied and J* is generated by any set of S'_{Mi} i = 1 to N, such that

$$5.35 \qquad \sum_{i}^{N} R'_{Fi} S'^{*}_{Mi} = \sum_{i}^{N} r^{*}_{Si} = r_L(1-\delta)$$

or for any single product i

$$5.36 \qquad S'^{*}_{Mi} = \frac{r_L(1-\delta) - \sum\limits_{j \neq i}^{N} R'_{Fj} S'^{*}_{Mj}}{R'_{Fi}} = \frac{r_L(1-\delta) - \sum\limits_{j \neq i}^{N} r^{*}_{Sj}}{p^{*}_{i}}$$

Note that from 4.8 $R'_{Fj} S'_{Mj} = r_{Sj} > 0$ for all j,

then

$$5.37 \qquad S'^{*}_{Mi} < S'^{*}_{Mi} = \frac{r_L(1-\delta)}{R'_{Fi}} = \frac{r_L(1-\delta)}{p^{*}_{i}} \quad \text{when N = 1 and i is}$$

the only product offerred by the bank.

Now, the desirable case when the bank offers only one product is characterized by $S_{Mi} \leq S^{*}_{i}$. Since $S_{Mi} = S'_{Mi} M$, in order to qualify for this case

$$5.38a \qquad S'^{*}_{Mi} = \frac{r_L(1-\delta)}{p^{*}_{i}} \leq \frac{S^{*}_{i}}{M^{**}} \quad \text{for N = 1 and i is the only product}$$

So when i is the only product, if 5.38a is not satisfied (ie. $S'^{*}_{Mi} = \frac{r_L(1-\delta)}{p^*_i} > \frac{S^*_i}{M^{**}}$ for N = 1, and we are in the troublesome case because $S_{Mi} > S^*_i$), the bank can offer additional products, each with S'^{*}_{Mj} low enough so that $S_{Mj} < S^*_j$. Now the condition necessary to reap the benefits described in Section IV (ie. when $S_{Mi} < S_i$) is

$$5.38b \qquad S'^{*}_{Mi} = \frac{r_L(1-\delta) - \sum\limits_{j \neq i}^{N} R'_{Fj} S'^{*}_{Mj}}{R'_{Fi}} = \frac{r_L(1-\delta) - \sum\limits_{j \neq i}^{N} r^*_{Sj}}{p^*_i} \leq \frac{S^*_j}{M^{**}}$$

Since $r_{Sj} > 0$ all j, and S^*_i and $M^{**} = \Phi(r_L\delta)$ are not effected by changes in N, as N increases S'^{*}_{Mi} decreases assuming we hold the other S'^{*}_{Mj} constant. Therefore, by offering additional products each with a low S'^{*}_{Mj} (so $S_{Mj} < S^*_j$), the bank can reduce S'^{*}_{Mi} an amount sufficient to satisfy 5.38b.

Hence, banks and firms can avoid the reduced joint benefit position resulting from a situation where 5.38a is not satisfied. The undesirable situation occurs when the quantity of some service product delivered with compensating is greater than the normal equilibrium quantity. The solution involves the bank's broadening its service product line which lowers the optimal rate at which a service product is supplied for balances, S'^{*}_{Mi}. Since $S_M = S'^{*}_{Mi}M^{**}$, this reduces the quantity of product i supplied through compensating balances.

By adding more products, S_M can be made to be less than S^*, the normal equilibrium quantity. Thus, even if $S_{Mi} > S_i^*$ when i is the only product, by increasing the number of products offered, the less attractive results (described in the first part of this section) can be avoided. Hence, the bank and the firm can achieve values of the implicit deposit rate, M^{**}, C^{**}, π^* and, most important, J^* equal to those in the case of an explicit cash deposit rate described in Section III.

This phenomenon explains the proliferation of the different services and products offered by banks in the United States. It implies that to result in the highest level of benefits, the compensating balance mechanism should never expand a firm's consumption of services beyond its normal equilibrium consumption resulting from direct cash payment. Also, the same rationale can explain the many and varied types of compensating balance arrangements between banks and firms (ie. concerning loans, lines of credit, as well as services). Such arrangements allow them to achieve the benefits of optimal compensating balances (described in Section IV) and circumvent the negative impact of the restriction in our economy which exludes explicit interest on demand deposits.

Table 5.1

Summary of Firm Money Demand with Compensating Balances in Payment for Services and $S_M > S^*$

$$S_M(M) = S'_M M \qquad R_F = (p^D(S_M))S_M < R_B = (p^S(S_M))S_M$$

$$R'_F = p^D + \frac{\partial p^D}{\partial S} S_M \quad \text{and} \quad R'_B = p^S + \frac{\partial p^S}{\partial S} S_M$$

Bank Maximization Yields: $S'^{*}_M = \dfrac{r_L(1-\delta)}{R'_B}$ and $r^*_S = R'_F S'^{*}_M = \dfrac{R'_F}{R'_B} r_L(1-\delta)$

Firm's Cost Minimization Condition: $-\dfrac{\partial T}{\partial M^{**}} = r_L - R'_F S'^{*}_M = r_L - r^*_S = r_L - \dfrac{R'_F}{R'_B} r_L(1-\delta)$

Yielding Optimal Firm Money Demand: $M^{**} = \Phi(-\dfrac{\partial T}{\partial M^{**}}) = \Phi(r_L - r^*_S) = \Phi(r_L - \dfrac{R'_F}{R'_B} r_L(1-\delta))$

Where Φ is the inverse of minus the marginal transaction cost function, $-\dfrac{\partial T}{\partial M}$

Optimal Firm Money Demand Yields:

Total Implicit Interest:

Firm's Viewpoint

$$=R_F(S_M(M^{**})) = \left(\frac{p^D}{p^S + \frac{\partial p^S}{\partial S} S_M}\right) (r_L(1-\delta))(\Phi(r_L - \frac{R'_F}{R'_B} r_L(1-\delta)))$$

Bank's Viewpoint

$$=R_B(S_M(M^{**})))=(\frac{p^S}{p^S + \frac{\partial p^S}{\partial S} S_M} X r_L(1-\delta))(\Phi(r_L - \frac{R'_F}{R'_B} r_L(1-\delta)))$$

Table 5.1 Continued

$$C^{**} = r_L M^{**} - R_F(S_M(M^{**})) + T(M^{**})$$

$$C^{**} = (r_L - (\frac{p^D}{p^S + \frac{\partial p^{S!}}{\partial S} S_M})(r_L(1-\delta)))(\Phi(r_L - \frac{R_F'}{R_B'} r_L(1-\delta))) + T(\Phi(r_L - \frac{R_F'}{R_B'} r_L(1-\delta))$$

$$\pi^* = r_L(1-\delta)M^{**} - R_B(S_M(M^{**}))$$

$$\pi^* = (r_L(1-\delta)(1 - \frac{p^S}{p^S + \frac{\partial p^S}{\partial S} S_M}))(\Phi(r_L - \frac{R_F'}{R_B'} r_L(1-\delta)))$$

$$J^* = -r_L \delta M^{**} - T(M^{**}) + R_F - R_B$$

$$J^* = \left((\frac{p^D - p^S}{p^S + \frac{\partial p^S}{\partial S} S_M})(r_L(1-\delta)) - r_L\delta)(\Phi(r_L - \frac{R_F'}{R_B'} r_L (1-\delta) \right) - T(\Phi(r_L - \frac{R_F'}{R_B'} r_L(1-\delta))$$

Table 5.1 Continued

Comparisons	No Compensating Balance Arrangement and $r_D=0$ (Table 4.1)		Compensating Balance arrangement with $S_M>S^*$ (Table 5.1)		Compensating Balance Arrangement with $S_M\leq S^*$ (Table 4.2)
Marginal Interest Rate on Deposits (Firm's Viewpoint)	$r_D=0$	$<$	$r_S^*=R_F'S_M'^*=\frac{R_F'}{R_B'}(r_L(1-\delta))$	$<$	r_S^*
Marginal Interest Rate on Deposits (Bank's Viewpoint)	$r_D=0$	$<$	$R_B'S_M'^* = r_L(1-\delta)$	$=$	r_S^*
Average Interest Rate on Deposits (Firm's Viewpoint)	$r_D=0$	$<$	$p^D S_M'^*$	§	r_S^*
Average Interest Rate on Deposits (Bank's Viewpoint)	$r_D=0$	$<$	$p^S S_M'^*$	§	r_S^*
Optimal Firm Money Demand	M**	$<$	M**	$<$	M**
Optimal Firm Cost	C**	$>$	C**	§	C**
Optimal Bank Profit	π^*	§	π^*	§	π^*
Optimal Bank-Firm Position	J*	§ J*	J*	$<$	J*

§ = indeterminant

Table 5.1 Continued

Compensating Balance Arrangements with:	Marginal Deposit Rate		Average Deposit Rate
$S_M \leq S^*$	$r_S^* = r_L(1-\delta)$	=	$r_S^* = r_L(1-\delta)$
$S_M > S^*$: Firm's Viewpoint	$r_S^* = R'_F S'_M{}^* = \frac{R'_F}{R'_B}(1-\delta)$	<	$p^D S'_M{}^*$
Bank's Viewpoint	$R'_B S'_M{}^*$	§	$p^S S'_M{}^*$

§ = indeterminant

VI. Conclusion

When the deposit rate is constrained to be less than the competitive rate (ie. $r_D < r_D = r_L(1-\delta)$), firms hold balances lower than the optimal money demand when r_D is competitively determined. ($M^{**} = \phi\ (r_L - r_D^*) = \phi\ (r_L\delta)$). This results in relatively high transaction costs as cash is managed intensively to keep deposits low. This creates an incentive for banks and firms to develop devices such as that discussed in section IV and V above which encourage firms to hold larger deposits. This paper demonstrates that, as balances are increased to M^{**}, the resulting savings in transaction cost is greater than the loss due to required reserves. In addition to the implications delineated in preceding sections of this paper, this analysis provides the foundation for the rationalization of the many such practices which encourage larger deposits (ie. compensating balances). The traditional analysis of these practices is unable to explain the holding of balances larger than the firm would hold in their absence. This results from the failure to account for the benefits from additional balances.[13]

The traditional analysis' inability to explain higher than normal balances conflicts with empirical studies. Such studies deomonstrate that firms hold inordinately large balances compared with theoretical expectations.

The analysis in this paper reconciles empirical findings with the optimization models of transactions demand theory. This

reconciliation involves the development of a rational theory of compensating balances which does explain the holding of higher balances by firms. Traditionally the money demand models (ie. Baumol-Tobin, Miller-Orr) incorporate a single rate (ie. the rate on loans or short term securities) as the opportunity cost to the firm of holding deposits. This is because it is assumed $r_D = 0$. If $r_D > 0$, then the spread between the traditional rate and the deposit rate (ie. $r_L - r_D$) would be the appropriate rate. The result would be much higher predicted holdings.

The analysis in this paper suggests that even when r_D is institutionally constrained to be zero, the optimizing behavior of firms and banks will result in an opportunity cost to firms of $r_L - r_S^* = r_L - r_D^* = r_L\delta$ (as opposed to r_L). It is this lower opportunity cost which accounts for higher balances; the same higher balances firms would hold in an economy with a competitively determined r_D.

Therefore, this study tends to reconcile empirical findings with the optimization models of transactions demand theory. This reconciliation involves the development of a rational theory of compensating balances.

FOOTNOTES

[1] This assumes that r_L is the same in both cases. A possible explanation for this practice is that the "artificial" loan demand (B) created results in increases in r_L sufficient to offset the dead weight loss described below. Even if the elasticity were such that this were true, the Fed, which presumably targets reserves with respect to the "real" loans and deposits (as opposed to "artificial" ones) which effect economic activity, would offset this effect by increasing reserves.

[2] More specifically the analysis above holds whenever the firm's rate, r_F, on securing B is related to the bank's lending (earning) rate, r_B, on $B(1 - \delta)$ such that $r_F > r_B (1 - \delta)$.

[3] I assume for convenience that the firm chooses to have a positive loan position. The other option is to raise long term funds and start with a desired short term position (ie. either $A = S^* + M$, desired short term earning assets plus desired demand deposits, or $A^* = S + M$ desired total liquidity) which is drawn down into loans as cash outflows require. The setting of the desired short term position (L^*, S^* or A^*) is not treated in this paper. Given this

position, however, the analysis holds in any case (e.g. with A^*, r_A the return on earning assets is the firm's opportunity cost on funds and the spread $r_L - r_A$ also enters the analysis without altering the results. An analogous adjustment is necessary if the firm's borrowing rate in C differs from the bank's average lending rate in π.) Both the conclusion that the compensating balance arrangement is rational and that money demand is unaffected by the restriction on r_D are valid in each of these frameworks.

[4] Time periods are defined as that period in which no adjusting inflows (or outflows) from (or to) long term sources occur. So the firm sets L^*(or A^* or S^*) and M using long sources (ie. if $L > L^* + M$, raises $L - (L^* + M)$ from long sources to reduce L to be equal to $L^* + M$; if $L < L^* + M$, increases bank loans $(L^* + M) - L$ and uses the funds for long purposes such as dividends, retiring debt, investment projects). The time period is from this setting of L at the level desired for short term liquidity until the next such adjustment involving long sources or uses. All costs, rates of return, profits etc, are defined on this time period.

[5] This general specification of T(M) subsumes the transaction cost functions of traditional models. In the Baumol model

$T(M) = b\frac{Y}{Q} = b\frac{Y}{\partial M}$ where b is the fixed (lumpy) transaction cost, Y income, Q desired begining cash and M the expected cash level. In the Miller-Orr model $T(M) = b\,P_T(M)$ where $P_T(M)$ is the daily probability of a transaction given the chosen expected deposit level. However, in this paper T(M) is very general in that it is only restricted by the assumptions listed above. The analysis is therefore valid not only for fixed (lumpy) transaction costs but also for proportional and mixed transaction cost functions.

[6] Note: in $\frac{\partial \pi}{\partial r_D} = (r_L(1-\delta) - r_D)\frac{\partial M}{\partial r_D} - M$, the first term is the rate of net revenue from the additional deposits generated by an increase in r_D; the second is the cost of paying additional r_D on the entire balance M. While in $\frac{\partial C^*}{\partial r_D} = (r_L - r_D)\frac{\partial M^*}{\partial r_D} + \frac{\partial T}{\partial M}\frac{\partial M^*}{\partial r_D} - M^* = -M^*$ (since at C^*, M^*, $\frac{\partial T}{\partial M^*} = -(r_L - r_D)$), so the opportunity cost on additional deposits is offset at the margin by savings in transaction cost, leaving a marginal reduction in cost equal to the additional r_D on M. Hence, $\frac{\partial J}{\partial r_D} = (r_L(1-\delta) - r_D)\frac{\partial M}{\partial r_D}$ since the cost to the bank and the return to the firm of additional interest payments cancels out, leaving only the net revenue (or loss) from additional deposits generated by the increase in r_D.

[7] For convenience I have assumed that the bank is willing to supply services for cash or balances, and the firm decides the quantity to demand through each mechanism. One could also assume that the bank is only willing to supply services for balances, but there is an outside firm (or bank) willing to supply services according to the $p^S(S)$ schedule. In this case the analysis is unchanged if and only if $\frac{\partial p^S}{\partial S} = 0$. If $\frac{\partial p^S}{\partial S} \neq 0$, then the analysis is altered since the firm is splitting its total demand for services between inside and outside suppliers which will alter the price it pays for services. This can be included in the analysis in a straightforward manner and will not alter the basic conclusion that compensating balances are rational. It will, however, effect the optimal money demand. It should be clear from the preceding section that these assumptions about the supply and demand for services are not crucial to the argument that compensating balances are rational. Their role is only in making explicit the implicit interest received when firm's receive services for deposits.

[8] This assumption implies that the price elasticity of demand $\equiv e > 1$, and therefore, the firm has an elastic demand for services over the relevant range (ie. $S = S_M > S^*$), since $\frac{\partial R_F}{\partial S_M} = p^D \left(\frac{1}{-e} + 1 \right)$. Actually, recalling that $r_S = R_F' S_M'$ it can be

shown that $\frac{\partial r_S}{\partial S'_M} > 0$ in spite of the fact that $\frac{\partial R'_F}{\partial S_M}$ may be negative. Furthermore, even if $\frac{\partial R'_F}{\partial S_M} < 0$ it can be shown that if $R'_F > 0$ over some small neighborhood $S_M = S^*+\varepsilon$, then increases in S'_M can never drive R'_F to zero (ie. the point of unitary elasticity) and as $S'_M \to \infty$, $r_S \to \infty$. This assures us that even if $\frac{\partial R'_F}{\partial M} < 0$, S'_M can force r_S high enough to equal any r^*_S required by optimality conditions. This is true even though $S_M > S^*$ is restricted to the elastic portion of the demand curve.

[9] When $S_M > S^*$, $\frac{\partial R_F}{\partial M} = p^D \frac{\partial S_M}{\partial M} \left(\frac{1}{-e} + 1\right) > 0$. So $r_S > 0$ only if the firm's demand is elastic at $S = S_M$.

[10] One could include this constraint analytically, of course, but there is clearly no such constraint in the business world.

[11] This loss is somewhat analogous to the loss of consumers and producer's surplus when the quantity demanded and supplied is below the equilibrium quantity.

[12] The firm itself is unequivocally better off with compensating balances since its cost is always lower. It is the bank's profit position which is indeterminate.

[13] That is the savings in transaction cost, which of course is the only reason firms hold positive demand deposits anyway. If one ignores such savings, firms and banks would agree that M should be zero, since every dollar borrowed from the bank to be deposited carries an opportunity cost to the firm $(r_L - r_D)$ and the bank $(r_L \delta)$.

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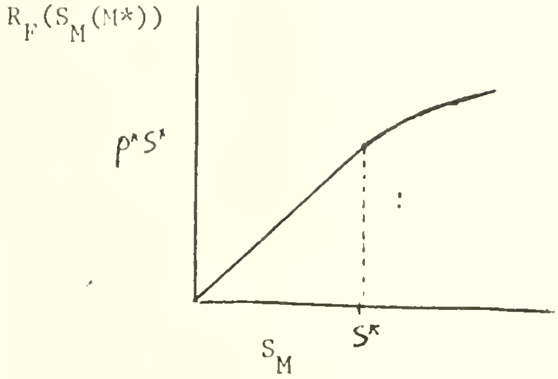
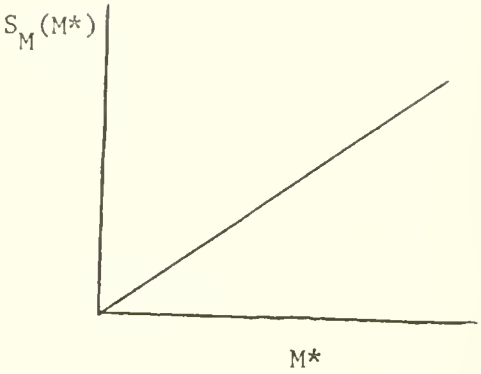
APPENDIX

Functions:





$$R_F = R_F(S_M(M)) = \begin{cases} p^*S_M & \text{for } S_M \leq S^* \\ (p^D(S_M))S_M & \text{for } S_M \geq S^* \end{cases}$$

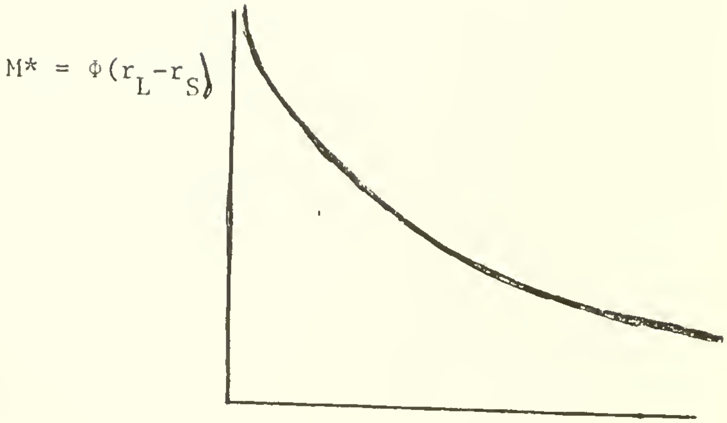
$$R_F' = \frac{\partial R_F}{\partial S_M} \qquad \frac{\partial R_F'}{\partial S_M} = \frac{\partial^2 R_F}{\partial S_M^2}$$

$$S_M = S_M(M^*)$$

$$S_M' = \frac{\partial S_M}{\partial M^*}$$

$$M^* = \Phi(r_L - r_S) = M^*(r_S)$$

$$r_S = r_S(R_F', S_M') = R_F'S_M'$$



$$-\frac{\partial T}{\partial M} = r_L - R_F'S_M' = r_L - r_S$$

from 4.13

$$R'_F\Big|_{S_M \le S^*} = p^* > 0 \qquad \frac{\partial R'_F}{\partial S_M}\Bigg|_{S_M \le S^*} = 0$$

$$R'_F\Big|_{S_M \ge S^*} = p^D + \frac{\partial p^D}{\partial S}\, S'_M > 0 \qquad \frac{\partial R'_F}{\partial S_M}\Bigg|_{S_M > S^*} \le 0 \quad \text{assumed}$$

assumed

Concerning the Bank's Decision Variable, $S_M(M)$, it is assumed,

$$S'_M > 0 \quad \text{and} \quad \frac{\partial S'_M}{\partial M^*} = 0$$

From 4.13 $M^* = \Phi(-\frac{\partial T}{\partial M}) = \Phi(r_L - r_S)$

where Φ is the inverse of minus the marginal transaction cost function, $-\frac{\partial T}{\partial M}$

$$\Phi' \equiv \frac{\partial \Phi}{\partial - \frac{\partial T}{\partial M^*}} = \frac{1}{-\frac{\partial^2 T}{\partial M^{*2}}} < 0 \quad \text{so by the chain rule}$$

$$\frac{\partial M^*}{\partial r_S} = \frac{1}{\frac{\partial^2 T}{\partial M^{*2}}} > 0 \text{ since it is assumed that } \frac{\partial^2 T}{\partial M^{*2}} > 0$$

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$r_S = R'_F S'_M = R'_F(S_M(M^*(r_S(R'_F, S'_M))))\ S'_M$

where the parentheses denote functions.

Therefore,

$$\frac{\partial r_S}{\partial S'_M} = R'_F + S'_M \frac{\partial R'_F}{\partial S'_M} = R'_F + S'_M \frac{\partial R'_F}{\partial S_M} \frac{\partial S_M}{\partial M^*} \frac{\partial M^*}{\partial r_S} \frac{\partial r_S}{\partial S'_M} \quad \text{so}$$

$$\frac{\partial r_S}{\partial S'_M} = \frac{R'_F}{1 - S'_M \frac{\partial R'_F}{\partial S_M} \frac{\partial S_M}{\partial M^*} \frac{\partial M^*}{\partial r_S}} = \frac{\frac{\partial R_F}{\partial S_M} \quad \frac{\partial^2 T}{\partial M^{*2}}}{\frac{\partial^2 T}{\partial M^{*2}} - \frac{\partial^2 R_F}{\partial S_M^2} \left(\frac{\partial S_M}{\partial M^*} \right)^2} > 0$$

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